Indian Institute of Technology Hyderabad

Department of Mathematics

Assignment 3

MA 4020 : Linear algebra   Max Marks: 50

1. Prove that if T^2 has a cyclic vector, then T has a cyclic vector. Is the converse true? Classify up to similarity all $n \times n$ complex matrices A such that $A^n = I$. [5]

2. Let T be a linear operator on the n-dimensional vector space V, and suppose that T has n-distinct characteristic values. Prove that T is diagonalizable. [5]

3. Suppose A is a 2×2 real symmetric matrix. Prove that A is similar over $\mathbb{R}$ to a diagonal matrix. [5]

4. Let N be a 2×2 complex matrix such that $N^2 = 0$. Prove that either $N = 0$ or N is similar over $\mathbb{C}$ to [5]
$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

5. Let A be the real matrix [5]
$$A = \begin{bmatrix} 3 & -4 & -4 \\ -1 & 3 & 2 \\ 2 & -4 & -3 \end{bmatrix}.$$
Write down the rational canonical form of A.

6. Let n be a positive integer, and let V be the space of polynomials over $\mathbb{R}$ which have degree at most n (throw in the 0-polynomial). Let D be the differential polynomial on V. What is the minimal polynomial for D? [5]

7. Let V be a finite-dimensional vector space and let $W_1, ..., W_k$ be subspaces of V such that [5]
$$V = W_1 + \cdots + W_k \text{ and } \dim V = \dim W_1 + \cdots + \dim W_k.$$
Prove that $V = W_1 \bigoplus \cdots \bigoplus W_k$.

8. Let A be a complex 5×5 matrix with characteristic polynomial [5]
$$f = (x-2)^3(x+7)^2$$
and the minimal polynomial $p = (x-2)^2(x+7)$. What is the Jordan form for A?

9. How many possible Jordan forms are there for a 6×6 complex matrix with characteristic polynomial $(x+2)^4(x-1)^2$? [5]

10. Find the Jordan form of A over $\mathbb{C}$, where A is [5]
$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$